"59 to 57 and returned after the eclipse nearly to its former "position."

I shall now proceed to give you the supplementary remarks which have been furnished by Professor Cleaveland.

"Our large *reflecting telescope* has the magnifying power of "450. I used the shortest eye-glass and middle-sized specu- "lum, which, if I am correct, magnifies 360 times.

"The President used his own telescope, and left the ma- "nagement of the large one to myself.—Its magnifying pow- "er is so great, that fearing lest I should not discover the com- "mencement of the eclipse, I kept the telescope in a slow mo- "tion, ranging backwards and forwards in a small arc. The "telescope was probably at one extremity of this arc, while "the immersion actually took place, for at the moment when "it was actually discovered by the telescope belonging to the "equatorial, I moved my telescope, and found the shadow "must have been discoverable two seconds at least. I allowed "one second for the motion of the telescope, after the eclipse "was seen by the observer with the equatorial, and the time "of the commencement was noted one second back accord- "ingly. This perfectly agreed with the observation of the "emersion.—We had some one at the clock, counting seconds; "and the shadow was visible one second longer by the large "telescope, than by the other, which circumstance was con- "sidered confirmatory of the allowance of one second made at "the commencement." So far the college observations extend.

I do not recollect to have heard of any accurate astronomi- cal observations, made in the United States to the north of Brunswick.

No. XLV.

On finding the longitude from the moon's meridian altitude, by William Dunbar of Natchez.

Read August 15th, 1806.

THE usual mode of making the lunar observation for the purpose of ascertaining the longitude, requires the aid of a

chronometer or good watch, to a single observer, and as time-pieces of a delicate construction are liable to derangement, the discovery of a method, by which one observer without a knowledge of the precise time, may be enabled to ascertain his longitude, becomes a desideratum of value.

There are two portions of time during each lunation, when the moon's change of declination is sufficiently rapid to afford the means of solving this useful problem; those times are, when the moon is on or near the celestial equator, and may be extended to four or five days at least, i. e. two before and two after the day on which the moon crosses the equator: the moon's change of declination in the most favourable circumstances, exceeds 6° in twenty-four hours, or 15″ in one minute of time and although this is scarcely half the moon's motion in longitude, yet it is to be remembered, that this method is no other than a meridian altitude, which may be taken to a degree of precision, never to be attained in the usual manner of taking the moon's distance from a star, and if the altitude be taken at land, with the aid of a mercurial horizon, the double angle will place this method on a footing of equality (nearly) with the usual mode, in respect to the moon's change of place, other circumstances being in its favour. The accuracy of this method, depends upon the correctness of the lunar meridian altitude, and the precision with which the latitude of the place of observation has been ascertained.

At sea, this method cannot always be used to advantage, on account of the ship's change of place, which might render the latitude doubtful to several minutes, and thereby affect the longitude an equal number of degrees.

The moon's greatest altitude being taken, a correction becomes necessary, because the greatest altitude is not on the meridian, but to the east or west, according as the moon is increasing or diminishing, by change of declination, her zenith distance, and which may be calculated as follows.—Having cleared the moon's apparent altitude from the effects of refraction and parallax, the difference between it and the co-latitude of the place of observation, will give the moon's declination nearly, from which by even proportion may be found, the ap-

proximate time at Greenwich: for this time find the rate of change of the moon's declination for one minute of time, and also the difference between the moon's meridian altitude, and its altitude one minute before or after, caused by the diurnal rotation of the earth, combined with the progress in right ascension; those two effects may be considered (at small distances from the meridian) as operating in the same line, and in opposite directions. Put x, to represent the time from the meridian, when the moon's altitude will be the same as when on the meridian, a, for the change in declination, and b, for the depression from the meridian in one minute of time; the first increases or diminishes in the direct, and the second (for small quantities) in the duplicate ratio of the times; hence we shall have $bx^2 = ax$, and therefore $x = \frac{a}{b}$; i. e. the time (in minutes) from the meridian, when the change in declination will be equalized by the depression, will be found by dividing the change of declination by the depression for one minute of time; at half this distance in time from the meridian, the change in declination will be double the depression, and will be the maximum, or point of greatest altitude, therefore $\frac{a^2}{4b}$ will represent the correction; i. e. the square of the rate of change of declination, divided by four times the depression for one minute of time, will be the correction for the moon's meridian altitude; which may be conveniently found by the following formula, expressed in logarithmic language.

RULE.

To twice the sine of 14′ 29″ 5 add the arith. comp. of the sine (or cosecant) of 1′ and the logarithm of 120, the sum, abating 20 from the index, will produce the constant logarithm .8651414. To the constant logarithm add the cosine of the declination of the moon, the cosine of the latitude, and the arith. comp. of the cosine (or secant) of the altitude, the sum, rejecting tens from the index, will be the logarithm of four times the depression, which subtracted from twice the loga-

rithm of the rate of change of declination for one minute of time, the remainder will be the logarithm of the correction in seconds.

EXAMPLE.

Given the moon's greatest altitude near the meridian, corrected from the effects of paral. and refraction, 45° 40′ 20″ 59; the latitude of the place, 32° 29′ 25″, and therefore the moon's declination (nearly) is 11° 50′ 14″ 41.

Constant logarithm. . . .	.8651414	Change in declination for 1′ of time	12″ 86
Decl. 11° 50′ 14″ 41 cosine.	9.9906648	Logarithm of ditto. . . .	1.1092412
			× 2
Lat. 32 29 25 cosine.	9.9260761	Square of change of decl.	Log. 2.2184820
Alt. 45 40 20 59 cos. ar. co.	.1556719	4 times the depression.	Log.—.9375542
Log. of 4 times the depression.	.9375542	Cor. for mer. alt. 19″ 096	Log. 1.2809278

The correction being found by the foregoing formula, is to be subtracted from the greatest altitude, cleared of the effects of refraction and parallax, the remainder will be the true altitude of the moon, when she was on the meridian. The difference between the corrected altitude of the moon's centre on the meridian and the colatitude, will be the moon's true declination, when on the meridian: the time at Greenwich, when the moon had that declination, being found, and also the time of the moon's transit over the meridian of Greenwich, take their difference, take also the difference of the increase of the A. R. of the ☽ and ☉ for the interval of time elapsed in passing the two meridians, which last difference being subtracted from the first difference, the remainder will be longitude in time.

In order to calculate with sufficient accuracy, the times corresponding to the moon's declination, and her transit over the meridian of Greenwich, it will be necessary to prepare four right ascensions and four declinations of the moon to seconds, which in the nautical almanac, are set down to minutes only, by the aid of which, with the tables of second differences, we can find very correctly the times which are sought, and in regard to the moon's declination, the effects of aberration and nutation should not be omitted, because an error of seconds in

the calculated declination, might produce an error of a like number of minutes of a degree in the longitude; those effects though of less importance in the moon's right ascension, may also be brought into calculation.

EXAMPLE I.

On the evening of the 10th of November, 1804, at Fort Miro, on the river Washita, took the apparent double altitude of the moon's lower limb (greatest) near the meridian, 89° 17′ 20″, index error + 13′ 47″ 5, the latitude of the place of observation 32° 29′ 25″. Required the longitude.

	°	′	″	
Double altitude of ☽'s lower limb.	89	17	20	
Index error	+	13	47	5
	2) 89	31	7	5
Apparent altitude of ☽'s lower limb.	44	45	33	75
Effects of refraction and parallax.	+	39	8	36
True altitude of ☽'s lower limb.	45	24	42	11
☽'s semidiameter and augmentation.	+	15	38	48
Altitude of ☽'s centre.	45	40	20	59
Correction by formula.		—	19	10
True altitude of ☽'s centre on meridian.	45	40	1	49
Colatitude.	57	30	34	91
☽'s declination on the meridian.	11	50	33	42

	″	
Rate of change of ☽'s declination in 1′ of time by even proportion.	13″	25
Correct by second differences.	— 0	39
True rate of change per minute at 12h 40′ Greenwich time.	12	86

	h	′	″	
Appt. time at Greenwich when the ☽ had this declination by even proportion.	12	39	56	17
Correct by the equation of second difference.	+	1	15	09
Apparent time at Greenwich when the ☽ was on the meridian of Fort Miro.	12	41	11	26
ditto. at ditto. when the ☽ was on the mer. of Greenwich interpolated.	6	22	39	22
Diff. of appt. time corrected by the equation of time —1″ 74 gives mean time.	6	18	30	3
Difference of increase of A. R. of the ☽ and ⊙ during the interval.		—11	41	
Longitude of Fort Miro.	6	6	49	3

Comparison of the above with other results.

	h	′	″	
Longitude deduced from a mean of six distances of the sun west of the moon.	6	5	59	
a mean of three distances of α Arietis east of ditto.	6	7	40	
Longitude. . . . . a lunar eclipse 14th of Jan. 1805, (a fine observation.)	6	6	42	5
Mean longitude of Fort Miro, differing from the result by the meridian altitude, only 2″ 13, or about 32″ of a degree.	6	6	47	17

EXAMPLE II.

October 7th, 1805, at the Forest plantation, latitude 31° 27′ 48″, observed the apparent double altitude of the moon's lower limb (greatest) near the meridian, 133° 11′ 14″

The index error being subtractive, add the lesser contact of the sun with his image taken immediately after observation. 15 30

		2)133 26 44
Lat. 31° 27′ 48″ .	Apparent altitude of the ☽'s centre.	66 43 22
Colat. 58 32 12 .	Parallax and refraction. . . .	+ 22 41 75
	True altitude of the ☽'s centre. .	67 6 3 75
	Correction per formula. . . .	—11 43
	True alt. of ☽'s centre on the mer.	67 5 52 32
	Colatitude.	58 32 12
	☽'s declination when on the meridian.	8 33 40 32

	h ′ ″
Appt. time at Greenwich when the ☽ had this declination by even proportion	17 43 52 18
Correction by the equation of second difference. .	— 2 12 47
Appt. time at Greenwich, when the ☽ was on the mer. of place of observation.	17 41 39 71
Appt. time corrected, when the ☽ was on the meridian of Greenwich. .	11 24 18 15
Difference of apparent time.	6 17 21 56
Correct for the equation of time.	+ 4 39
Difference in mean time, of the ☽ passing the two meridians. . . .	6 17 25 95
Difference of A. R. of the ☽ and ☉, gained during the interval. . .	—12 5
Longitude of the place of observation.	6 5 20 95

Mr. Ellicott has made 30 calculations, on which he seems to rely for the longitude of the Natchez, (others were rejected,) his extreme results are 6h 4′ 27″ to 6h 6′ 41″, and a mean of the whole is 6h 5′ 49″. The position of the Forest plantation is about 2½ miles east of Natchez, i. e. 9″ in time, which being added to the above result, gives 6h 5′ 30″ for the longitude of Natchez, differing from the mean of Mr. Ellicott's observations 19″ or 4¾ miles.

An immersion of Jupiter's 1st satellite was taken just before his opposition, and an emersion of the same soon after, and as they were probably both affected by the near vicinity of the light of Jupiter's disk, but acting in contrary directions, a mean of the two results may be supposed near to the truth, subject to the correction which the Greenwich calculations may require.

June 11th. By an immersion of Jupiter's 1st satellite. . . .	6h 5′ 41″ 4
July 6th an emersion of ditto.	6 5 12 19
Mean longitude.	6 5 26 8

The mean differs only 5″ 8, nearly 1⅓ mile from the result of the meridian altitude.

☞ In the above method of finding the longitude, as a small error in the meridian altitude of the moon, will produce a considerable one in the longitude, a correction ought to be applied on account of the spheroidal figure of the earth............EDIT.